

# Squeezed Neutrino Oscillations

## in Quantum Field Theory

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### Abstract

By resorting to recent results on fermion mixing which show that the Fock space of definite flavor states is unitarily inequivalent to the Fock space of definite mass states, we discuss the phenomenological implications on the neutrino oscillation formula. For finite momentum the oscillation amplitude is depressed, or "squeezed", by a momentum dependent factor. In the relativistic limit the conventional oscillation formula is recovered.

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In a recent paper [1] the mixing transformations of fermion fields have been studied in the framework of the Lehmann-Symanzik-Zimmermann (LSZ) formalism of quantum field theory (QFT) [2] and particular attention has been devoted to the mixing transformations of massive Dirac neutrino fields in view of their relevance to neutrino physics and related questions of great interest to cosmology as well as to solar physics and modelling [3].

The analysis presented in ref. 1 shows that much care is needed in the identification of the proper vacuum state for the mixed fields since the Fock space for the original (free) fields turns out to be unitarily inequivalent to the Fock space for the mixed fields in the infinite volume limit.

Difficulties in defining the appropriate creation and annihilation operators for mixed neutrino fields such as the ones which are used in the standard treatment of neutrino oscillations were already pointed out in [4]. There it was shown that it is in fact impossible to construct operators for weak states which obey canonical anticommutation relations. "Approximate" operators and the corresponding "approximate Fock space" were constructed which were shown to exist only in the relativistic limit and for almost degenerate mass eigenvalues. In the light of the results presented in ref. 1 it now appears that the difficulties pointed out in ref. 4 may find their origin in the unitary inequivalence between the mixed fields and the massive (free) fields Fock spaces. Creation and annihilation operators for the mixed fields which satisfy canonical anticommutation relations are explicitly constructed in ref. 1 and the vacuum state for a well definite Fock space is found to be an  $SU(2)$  generalized coherent state with neutrino-antineutrino condensate structure.

The results of ref. 1, when applied to neutrino mixing, lead to non-trivial consequences in the neutrino oscillation formula, as we will explain below. The question then arises if and to which extent such results may change the experimental expectations and may be eventually tested. The purpose of this paper is indeed to discuss such a question and to estimate the corrections to the conventional neutrino oscillation formula.

For the reader convenience, let us briefly summarize the results of ref. 1. We

will omit all the mathematical analysis and derivations which are there reported in detail.

We consider the Pontecorvo mixing relations [5] (for simplicity we confine ourselves to two flavors; for the case of three flavors see ref. 1):

$$\begin{aligned}\nu_e(x) &= \nu_1(x) \cos \theta + \nu_2(x) \sin \theta \\ \nu_\mu(x) &= -\nu_1(x) \sin \theta + \nu_2(x) \cos \theta ,\end{aligned}\tag{1}$$

where  $\nu_e(x)$  and  $\nu_\mu(x)$  are the (Dirac) neutrino fields with definite flavors.  $\nu_1(x)$  and  $\nu_2(x)$  are the (free) neutrino fields with definite masses  $m_1$  and  $m_2$ , respectively. Here we do not need to distinguish between left-handed and right-handed components. The fields  $\nu_1(x)$  and  $\nu_2(x)$  are written as

$$\nu_i(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}, r} [u_{\vec{k}, i}^r \alpha_{\vec{k}, i}^r e^{i\vec{k} \cdot \vec{x}} + v_{\vec{k}, i}^r \beta_{\vec{k}, i}^{r\dagger} e^{-i\vec{k} \cdot \vec{x}}], \quad i = 1, 2 .\tag{2}$$

$\alpha_{\vec{k}, i}^r$  and  $\beta_{\vec{k}, i}^r$ ,  $i = 1, 2$ ,  $r = 1, 2$  are the annihilator operators for the vacuum state  $|0\rangle_{1,2}$ :  $\alpha_{\vec{k}, i}^r |0\rangle_{12} = \beta_{\vec{k}, i}^r |0\rangle_{12} = 0$ . Here and in the following, as far as no misunderstanding arises, we omit time dependence. The anticommutation relations are:

$$\{\nu_i^\alpha(x), \nu_j^{\beta\dagger}(y)\}_{t=t'} = \delta^3(x - y) \delta_{\alpha\beta} \delta_{ij}, \quad \alpha, \beta = 1, \dots, 4 ,\tag{3}$$

and

$$\{\alpha_{\vec{k}, i}^r, \alpha_{\vec{q}, j}^{s\dagger}\} = \delta_{kq} \delta_{rs} \delta_{ij}; \quad \{\beta_{\vec{k}, i}^r, \beta_{\vec{q}, j}^{s\dagger}\} = \delta_{kq} \delta_{rs} \delta_{ij}, \quad i, j = 1, 2 .\tag{4}$$

All other anticommutators are zero. The orthonormality and completeness relations are the usual ones.

Eqs.(1) relate the hamiltonians  $H_{1,2}$  (we consider only the mass terms) and  $H_{e,\mu}$  [5]:

$$H_{1,2} = m_1 \bar{\nu}_1 \nu_1 + m_2 \bar{\nu}_2 \nu_2\tag{5}$$

$$H_{e,\mu} = m_{ee} \bar{\nu}_e \nu_e + m_{\mu\mu} \bar{\nu}_\mu \nu_\mu + m_{e\mu} (\bar{\nu}_e \nu_\mu + \bar{\nu}_\mu \nu_e)\tag{6}$$

where  $m_{ee} = m_1 \cos^2 \theta + m_2 \sin^2 \theta$ ,  $m_{\mu\mu} = m_1 \sin^2 \theta + m_2 \cos^2 \theta$  and  $m_{e\mu} = (m_2 - m_1) \sin \theta \cos \theta$ .

It is useful to mention at this point that in the LSZ formalism of QFT [2] asymptotic in- (or out-) fields, also called free or physical fields, in terms of which observables are expressed, are obtained by the weak limit of the Heisenberg or interacting fields for  $t \rightarrow -(or+)\infty$ . The basic dynamics, namely the system Lagrangian and the resulting field equations, is given in terms of the Heisenberg fields and therefore the meaning of the weak limit is to provide a realization of the basic dynamics in terms of the asymptotic fields. Such a realization, i.e. the weak limit, is however not unique since infinitely many representations of the canonical (anti-)commutation relations exist in QFT [2, 6, 7]. Well known examples of such a situation are the theories where spontaneous breakdown of symmetry is possible. There, the same set of Heisenberg fields and the same basic dynamics can be realized by asymptotic limit in the normal (symmetric) phase as well as in the broken symmetry phase. Therefore, since unitarily inequivalent representations describe physically different phases, in order to avoid ambiguities, it is of crucial importance to investigate with much care the mapping among Heisenberg fields and free fields (these mappings are usually called dynamical mappings or Haag expansions [6, 7]).

The above remarks apply to QFT, where systems with infinite number of degrees of freedom are considered. In quantum mechanics, namely for finite volume systems, the von Neumann theorem ensures that the representations of the canonical commutation relations are each other unitary equivalent and no problem arises with uniqueness of the asymptotic limit. However, the von Neumann theorem does not hold in QFT and much attention is required when considering any mapping among interacting and free fields [6, 7].

For these reasons, intrinsic to the QFT structure, mixing relations such as the relations (1) deserve a careful analysis.

It was in fact the purpose of ref. 1 to investigate the structure of the Fock spaces  $\mathcal{H}_{1,2}$  and  $\mathcal{H}_{e,\mu}$  relative to  $\nu_1(x)$ ,  $\nu_2(x)$  and  $\nu_e(x)$ ,  $\nu_\mu(x)$ , respectively. In particular, there it was indeed shown that the massive fields space  $\mathcal{H}_{1,2}$  and the flavor fields space  $\mathcal{H}_{e,\mu}$  become orthogonal (i.e. unitarily inequivalent) in the infinite volume limit:  $\lim_{V \rightarrow \infty} {}_{1,2}\langle 0|0\rangle_{e,\mu} = 0$ , where  $|0\rangle_{e,\mu}$  denotes the vacuum for the flavor field

operators. This is an exact result in QFT and is a novel feature with respect to the conventional treatment of neutrino mixing.

The unitary inequivalence in the infinite volume limit of the mass and the flavor representations shows the absolutely non-trivial nature of the mixing transformations (1). In fact, one can show that the mixing transformations induce a physically non-trivial structure in the flavor vacuum state which indeed turns out to be an  $SU(2)$  generalized coherent state [8] exhibiting neutrino-antineutrino pair condensation [1].

We thus realize the limit of validity of the approximation usually adopted when the vacuum state of the representation for definite mass operators is identified with the vacuum state for the flavor operators. We point out that even at finite volume the vacua identification is actually an approximation since the flavor vacuum is an  $SU(2)$  generalized coherent state. In such an approximation, the coherent state structure with pair condensation is in fact missed.

In conclusion, only in a theoretically rude approximation one may assume that massive neutrino fields and flavor neutrino fields share the same vacuum state and the same Fock space representation. The problem is, however, to see if the proper theoretical treatment leads to any interesting and testable effect out of reach in the heuristic conventional approximation. Our following discussion is aimed to such a task.

Without loss of generality, one can choose [1] the reference frame such that  $k = (0, 0, |k|)$ . The mixing transformations (1) then lead to the mappings in terms of creation and annihilation operators [1]:

$$\alpha_{k,e}^r = \cos \theta \alpha_{k,1}^r + \sin \theta \left( U_k^* \alpha_{k,2}^r + \epsilon^r V_k \beta_{-k,2}^{r\dagger} \right) \quad (7a)$$

$$\alpha_{k,\mu}^r = \cos \theta \alpha_{k,2}^r - \sin \theta \left( U_k \alpha_{k,1}^r - \epsilon^r V_k \beta_{-k,1}^{r\dagger} \right) \quad (7b)$$

$$\beta_{-k,e}^r = \cos \theta \beta_{-k,1}^r + \sin \theta \left( U_k^* \beta_{-k,2}^r - \epsilon^r V_k \alpha_{k,2}^{r\dagger} \right) \quad (7c)$$

$$\beta_{-k,\mu}^r = \cos \theta \beta_{-k,2}^r - \sin \theta \left( U_k \beta_{-k,1}^r + \epsilon^r V_k \alpha_{k,1}^{r\dagger} \right) \quad (7d)$$

with  $\epsilon^r = (-1)^r$  and

$$V_k = |V_k| e^{i(\omega_{k,2} + \omega_{k,1})t} \quad , \quad U_k = |U_k| e^{i(\omega_{k,2} - \omega_{k,1})t} \quad (8)$$

$$|U_k| = \left( \frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{\frac{1}{2}} \left( \frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{\frac{1}{2}} \left( 1 + \frac{k^2}{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)} \right) \quad (9a)$$

$$|V_k| = \left( \frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{\frac{1}{2}} \left( \frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{\frac{1}{2}} \left( \frac{k}{(\omega_{k,2} + m_2)} - \frac{k}{(\omega_{k,1} + m_1)} \right) \quad (9b)$$

$$|U_k|^2 + |V_k|^2 = 1 \quad (10)$$

$$|V_k|^2 = |V(k, m_1, m_2)|^2 = \frac{k^2 [(\omega_{k,2} + m_2) - (\omega_{k,1} + m_1)]^2}{4 \omega_{k,1} \omega_{k,2} (\omega_{k,1} + m_1) (\omega_{k,2} + m_2)} \quad (11)$$

where  $\omega_{k,i} = \sqrt{k^2 + m_i^2}$ .

Most part of our discussion in this paper will be focused on the function  $|V_k|^2$ .

We notice that from eqs.(7) the expectation value of the number operator  $N_{\sigma_l}^{k,r}$  is obtained as:

$${}_{1,2}\langle 0 | N_{\sigma_l}^{k,r} | 0 \rangle_{1,2} = \sin^2 \theta |V_k|^2, \quad \sigma = \alpha, \beta, \quad l = e, \mu, \quad (12)$$

in contrast with the usual approximation case where one puts  $|0\rangle_{e,\mu} = |0\rangle_{1,2} \equiv |0\rangle$  and it is  $\langle 0 | N_{\alpha_e}^{k,r} | 0 \rangle = \langle 0 | N_{\alpha_\mu}^{k,r} | 0 \rangle = 0$ . Eq.(12) gives the condensation density of the vacuum state  $|0\rangle_{1,2}$  as a function of the mixing angle  $\theta$ , of the masses  $m_1$  and  $m_2$ , and of the momentum modulus  $k$ . It has been also observed that  ${}_{1,2}\langle 0 | N_{\sigma_l}^{k,r} | 0 \rangle_{1,2}$  plays the role of zero point contribution when considering the energy contribution of  $\sigma_l^{k,r}$  particles [1].

The oscillation formula is finally obtained by using the mixing mappings (7) [1]:

$$\begin{aligned} & \langle \alpha_{k,e}^r(t) | N_{\alpha_e}^{k,r} | \alpha_{k,e}^r(t) \rangle = \\ & = 1 - \sin^2 \theta |V_k|^2 - |U_k|^2 \sin^2 2\theta \sin^2 \left( \frac{\Delta\omega_k}{2} t \right). \end{aligned} \quad (13)$$

The fraction of  $\alpha_\mu^{k,r}$  particles in the same state is

$$\begin{aligned} & \langle \alpha_{k,e}^r(t) | N_{\alpha_\mu}^{k,r} | \alpha_{k,e}^r(t) \rangle = \\ & = |U_k|^2 \sin^2 2\theta \sin^2 \left( \frac{\Delta\omega_k}{2} t \right) + \sin^2 \theta |V_k|^2 (1 - \sin^2 \theta |V_k|^2). \end{aligned} \quad (14)$$

The occurrence of  $|V_k|^2$  and of  $|U_k|^2$  in (13) and (14) denote the contribution from the vacuum condensate. We observe that

$$\begin{aligned} \langle \alpha_{k,e}^r(t) | N_{\alpha_e}^{k,r} | \alpha_{k,e}^r(t) \rangle + \langle \alpha_{k,e}^r(t) | N_{\alpha_\mu}^{k,r} | \alpha_{k,e}^r(t) \rangle = \\ \langle \alpha_{k,e}^r | N_{\alpha_e}^{k,r} | \alpha_{k,e}^r \rangle + \langle \alpha_{k,e}^r | N_{\alpha_\mu}^{k,r} | \alpha_{k,e}^r \rangle . \end{aligned} \quad (15)$$

where  $|\alpha_{k,e}^r\rangle = |\alpha_{k,e}^r(t=0)\rangle$ , which shows the conservation of the number  $(N_{\alpha_e}^{k,r} + N_{\alpha_\mu}^{k,r})$ . Notice that the expectation value of this number in the state  $|0\rangle_{1,2}$  is not zero due to the condensate contribution.

Eqs.(13) and (14) are to be compared with the approximated ones in the conventional treatment:

$$\langle \alpha_{k,e}^r(t) | N_{\alpha_e}^{k,r} | \alpha_{k,e}^r(t) \rangle = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta\omega_k}{2} t \right) \quad (16)$$

and

$$\langle \alpha_{k,e}^r(t) | N_{\alpha_\mu}^{k,r} | \alpha_{k,e}^r(t) \rangle = \sin^2 2\theta \sin^2 \left( \frac{\Delta\omega_k}{2} t \right) , \quad (17)$$

respectively.

The QFT results (13) and (14) reproduces the conventional ones (16) and (17) when  $|U_k| \rightarrow 1$  (and  $|V_k| \rightarrow 0$ ).

In conclusion, eqs.(13) and (14) exhibit the corrections to the flavor oscillations coming from the condensate contributions. The conventional (approximate) results (16) and (17) are obtained when the condensate contributions are missing (in the  $|V_k| \rightarrow 0$  limit).

To discuss the phenomenological implications of the results (13) and (14) we have to study the function  $|V_k|^2$ .

Let us immediately observe that  $|V_k|^2$  depends on  $k$  only through its modulus and it is always in the interval  $[0, \frac{1}{2}]$ . It has a maximum for  $k = \sqrt{m_1 m_2}$ . Also,  $|V_k|^2 \rightarrow 0$  when  $k \rightarrow \infty$ . Moreover,  $|V_k|^2 = 0$  when  $m_1 = m_2$  (no mixing occurs in Pontecorvo theory in this case).

It is remarkable that the corrections to the oscillations depend on the modulus  $k$  through  $|V_k|^2$  (and  $|U_k|^2 = 1 - |V_k|^2$ ). Since  $|V_k|^2 \rightarrow 0$  when  $k \rightarrow \infty$ , these

corrections disappear in the infinite momentum or relativistic ( $k \gg \sqrt{m_1 m_2}$ ) limit. However, for finite  $k$ , the oscillation amplitude is depressed, or "squeezed", by a factor  $|U_k|^2$ : the squeezing factor ranges from 1 to  $\frac{1}{2}$  depending on  $k$  and on the masses values. The values of the squeezing factor may therefore have not negligible effects in experimental findings and the dependence of the flavor oscillation amplitude on the momentum could thus be tested.

To better estimate the effects of the momentum dependence it is useful to rewrite the  $|V_k|^2$  function as

$$|V_k|^2 \equiv |V(p, a)|^2 = \frac{1}{2} \left( 1 - \frac{(p^2 + 1)}{\sqrt{(p^2 + 1)^2 + a p^2}} \right) = \frac{1}{2} \left( 1 - \frac{1}{\sqrt{1 + a \left( \frac{p}{p^2 + 1} \right)^2}} \right) \quad (18)$$

with

$$p = \frac{k}{\sqrt{m_1 m_2}} \quad , \quad a = \frac{(\Delta m)^2}{m_1 m_2} \quad , \quad 0 \leq a < +\infty \quad , \quad (19)$$

where  $\Delta m \equiv m_2 - m_1$  (we take  $m_1 \leq m_2$ ).

At  $p = 1$ ,  $|V(p, a)|^2$  reaches its maximum value  $|V(1, a)|^2$ , which goes asymptotically to  $1/2$  when  $a \rightarrow \infty$ .

It is useful to calculate the value of  $p$ , say  $p_\epsilon$ , at which the function  $|V(p, a)|^2$  becomes a fraction  $\epsilon$  of its maximum value  $V(1, a)$ . This can be obtained by solving the equation:

$$|V(p_\epsilon, a)|^2 = \epsilon |V(1, a)|^2 \quad . \quad (20)$$

The solution of this equation is

$$p_\epsilon = \sqrt{-c + \sqrt{c^2 - 1}} \quad , \quad c \equiv \frac{b^2(a + 2) - 2}{2(b^2 - 1)} \quad , \quad b \equiv 1 - \epsilon \left( 1 - \frac{2}{\sqrt{a + 4}} \right) \quad . \quad (21)$$

In Tab. 1 are reported the values of  $\sqrt{m_1 m_2}$  and of  $a$  corresponding to some given values of  $m_1$  and  $m_2$  chosen below the current experimental bounds.

In Tab. 2 three sets of values of  $|U(p_\epsilon, a)|^2$  and of  $k_\epsilon$ , for  $\epsilon = 1, \frac{1}{2}, \frac{1}{10}$ , corresponding to the values of  $m_1$  and  $m_2$  given in Tab. 1, are reported (see also Fig. 1). We used  $|U(p_\epsilon, a)|^2 = 1 - \epsilon + \epsilon |U(1, a)|^2$  and  $k_\epsilon = p_\epsilon \sqrt{m_1 m_2}$ .

We see that for neutrinos of not very large momentum sensible squeezing factors for the oscillation amplitudes may be obtained. Larger deviations from the usual oscillation formula may thus be expected in these low momentum ranges. We note that observations of neutrino oscillations by large passive detectors include neutrino momentum as low as few hundreds of KeV [3].

We observe that the functional dependence of the oscillating amplitude on the momentum is such that, if experimentally tested, may give an indication on the neutrino masses since the function  $|U_k|^2$  (cf. eqs.(10) and (13)) has a minimum at  $k = \sqrt{m_1 m_2}$ .

It is interesting to observe that, although in a different framework where the neutrino wave packet spreading and the effect of spatially localized source and detectors were studied, also in refs. 9 it has been pointed out that non relativistic neutrinos with different masses are expected to give rise to drastically depressed oscillation amplitudes, the usual oscillation formula being recovered in the relativistic limit. As a conclusion, probing the non relativistic momentum domain seems therefore promising in order to obtain new insights in neutrino physics.

Since, as we have shown, the correction factor is related to the vacuum condensate, we see that the vacuum acts as a "momentum (or spectrum) analyzer" for the oscillating neutrinos: neutrinos with  $k \gg \sqrt{m_1 m_2}$  have oscillation amplitude larger than neutrinos with  $k \simeq \sqrt{m_1 m_2}$ , due to the vacuum structure. Such a vacuum spectral

Table 1:

	$m_1(eV)$	$m_2(KeV)$	$\sqrt{m_1 m_2}(KeV)$	$a$
<i>A</i>	5	250	1.12	$\sim 5 \cdot 10^4$
<i>B</i>	2.5	250	0.79	$\sim 1 \cdot 10^5$
<i>C</i>	5	200	1	$\sim 4 \cdot 10^4$
<i>D</i>	1	100	0.32	$\sim 1 \cdot 10^5$
<i>E</i>	0.5	50	0.15	$\sim 1 \cdot 10^5$
<i>F</i>	0.5	1	0.02	$\sim 2 \cdot 10^3$

Table 2:

	$ U(1, a) ^2$	$k_1(KeV)$	$ U(p_{1/2}, a) ^2$	$k_{1/2}(KeV)$	$ U(p_{1/10}, a) ^2$	$k_{1/10}(KeV)$
$A$	$\simeq 0.5$	1.12	$\simeq 0.75$	$\simeq 146$	$\simeq 0.95$	$\simeq 519$
$B$	$\simeq 0.5$	0.79	$\simeq 0.75$	$\simeq 145$	$\simeq 0.95$	$\simeq 518$
$C$	$\simeq 0.5$	1	$\simeq 0.75$	$\simeq 117$	$\simeq 0.95$	$\simeq 415$
$D$	$\simeq 0.5$	0.32	$\simeq 0.75$	$\simeq 58$	$\simeq 0.95$	$\simeq 206$
$E$	$\simeq 0.5$	0.16	$\simeq 0.75$	$\simeq 29$	$\simeq 0.95$	$\simeq 104$
$F$	$\simeq 0.5$	0.02	$\simeq 0.75$	$\simeq 0.6$	$\simeq 0.95$	$\simeq 2$

analysis effect may sum up to other effects (such as MSW effect [10] in the matter; in this connection we observe that the above scheme is easily generalized to the oscillations in the matter, see ref.1) in depressing or enhancing neutrino oscillations.

Finally, we remark that, as shown in ref. 1, the ratio of the amplitudes of the  $|\alpha_{k,1}{}^r\rangle$  and  $|\alpha_{k,2}{}^r\rangle$  components of the state  $|\alpha_{k,e}{}^r(t)\rangle$  is constant in time and that such a feature persists even in the relativistic limit, where, however, the oscillation formula reduces to the usual one. This reminds us of the SU(2) coherent state structure of the vacuum state [1] and is in contrast with the conventional treatment where the phase factor  $\exp(-i\Delta\omega t)$  produces "decoherence" between the components  $|\alpha_{k,1}{}^r\rangle$  and  $|\alpha_{k,2}{}^r\rangle$ .

In conclusion, the above discussion shows that the momentum dependence of the oscillation amplitude may be subject to experimental test and may provide novel features with respect to the conventional treatment of neutrino mixing. The vacuum condensate structure may manifest itself through its phenomenological consequences on the neutrino oscillations.

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## Figure caption

Fig. 1: The function  $|U(p, a)|^2$ .

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